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IDENTIFICATION OF NONLINEAR DELAY SYSTEMS USING SPLINE METHODS*

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H.T. Banks

Abstract

Spline based approximation schemes for nonlinear nonautonomous functional differential equation control systems are discussed and it is shown how these may be employed in parameter estimation techniques. A sample of our numerical findings is also presented.

1. Introduction

In this note we present results for a general class of spline approximations for nonlinear functional differential equations (FDE). The results of §2 below extend to a broad class of nonlinear FDE the basic state approximation theorems based on spline methods developed earlier for linear FDE in [3]. Our methods here do not employ a Trotter-Kato type theorem (the results for linear systems in [3] as well as those for autnomomous nonlinear FDE in [6] are based on Trotter-Kato approximation theorems). Rather, we use only simple ideas (Gronwall's inequality in a manner similar to that in [1] where another class of approximations for nonlinear control systems is developed) involving a dissipative condition on the nonlinear operator generated by the right-hand side of our nonlinear FDE. (For other spline approximation results for nonlinear FDE, see [8], [9].)

The ideas developed here are sketched for a specific class ("first-order" or "piecewise linear" splines) of approximations, but a careful reading of [3] in conjunction with analysis of our presentation below should convince the reader that our results extend immediately to classes of higher-order spline methods based on the approximation scheme $A^N = P^N A P^N$ used in [3] and below.

The state approximation results of §2 are given under a global Lipschitz hypothesis on the system that is sufficiently weak so as to include many systems arising in applications as special cases. Our results can be developed (at the expense of considerable added technical argument) under somewhat weaker conditions based on local Lipschitz criteria in certain of the arguments appearing in the right-hand side of our FDE system. We do include in our treatment nonautonomous systems such as those commonly found in control and identification problems. In fact, in §3 below we indicate how our results apply

directly to parameter identification problems and present some of our numerical findings. The applications of the approximation theorems of §2 to the nonlinear system identification problems of §3 are analogues to the linear system spline approximation identification techniques discussed in [2].

In obtaining our state approximation results (along with error estimates) in §2 we require a differentiability assumption on the right-hand side of the FDE in addition to the global Lipschitz hypothesis. This appears essential for our approach, which relies on convergence of the approximating operators A^N along solutions of the FDE (this in turn requires certain minimal smoothness criteria on trajectories). In contrast, the autonomous Trotter-Kato nonlinear approximation result developed in [6] requires only the global Lipschitz hypothesis. Of course, to provide error estimates, that approach must also include the additional differentiability condition.

Throughout we shall use the notation H^j or $H^j(a,b)$ to denote the usual Sobolev spaces $W_2^{(j)}(a,b;R^n)$ of R^n-valued functions, with $H^0 = L_2(a,b)$.

2. State Approximation for Nonlinear Systems

We consider the system

$$\begin{aligned} (2.1)\qquad & \dot{x}(t) = f(x(t),x_t,x(t-\tau_1), \ldots, x(t-\tau_\nu)) + g(t), \qquad 0 \le t \le T, \\ & x_0 = \phi \end{aligned}$$

where $f = f(\eta,\psi,y_1,\ldots,y_\nu): Z \times R^{n\nu} \to R^n$. Here $Z = R^n \times L_2(-r,0;R^n)$, $0 < \tau_1 < \ldots < \tau_\nu = r$, x_t denotes the usual function $x_t(\theta) = x(t+\theta)$, $-r \le \theta \le 0$, and $\phi \in H^1(-r,0)$. We shall make use of the following hypotheses throughout our presentation.

(H1): The function f satisfies a global Lipschitz condition:

$$|f(\eta,\psi,y_1,\ldots,y_\nu)-f(\xi,\tilde{\psi},w_1,\ldots,w_\nu)| \leq K\{|\eta-\xi|+|\psi-\tilde{\psi}| + \sum_{i=1}^{\nu}|y_i-w_i|\}$$

for some fixed constant K and all $(\eta,\psi,y_1,\ldots,y_\nu)$, $(\xi,\tilde{\psi},w_1,\ldots,w_\nu)$ in $Z \times R^{n\nu}$.

(H2): The function $f\colon Z \times R^{n\nu} \to R^n$ is differentiable.

Remark 2.1. If we define the function $F\colon R^n \times C(-r,0;R^n) \subset Z \to R^n$ given by

$$(2.2) \qquad F(z) = F(\eta,\psi) = f(\eta,\psi,\psi(-\tau_1),\ldots,\psi(-\tau_\nu))$$

we observe that even though f satisfies (H1), F will not satisfy a continuity hypothesis on its domain in the Z norm.

We define the nonlinear operator $A\colon D(A) \subset Z \to Z$ by

$$D(A) = W \equiv \{(\psi(0),\psi)\,|\,\psi \in H^1(-r,0)\}$$

$$A(\psi(0),\psi) = (F(\psi(0),\psi),\ D\psi)$$

where here $D\psi = \psi'$.

Theorem 2.1. Assume that (H1) holds and let $y(t;\phi,g) = (x(t;\phi,g),x_t(\phi,g))$ where x is the solution of (2.1) corresponding to $\phi \in H^1$, $g \in L_2$. Then for $\zeta = (\phi(0),\phi)$, $y(\phi,g)$ is the unique solution on $[0,T]$ of

$$(2.3) \qquad z(t) = \zeta + \int_0^t \{Az(\sigma)+(g(\sigma),0)\}d\sigma.$$

Furthermore, $g \to z(t;\phi,g)$ is weakly sequentially continuous from L_2 (weak) to Z (strong).

Our proof of Theorem 2.1 follows ideas of D. Reber (see [10] and §2, §3 of [11]) and employs a very general fixed-point theorem (a "uniform contraction" result - see [5, p. 7] - that is by now well known to those working in dynamical systems) to establish existence and continuous dependence of solutions of (2.3). For existence, uniqueness and continuous dependence of solutions of (2.1), we note that our condition (H1) is a global version of the hypothesis of Kappel and Schappacher in [7], so that their results yield immediately the desired result for (2.1).

The uniqueness of solutions to (2.3) follows in the usual manner once we establish that A satisfies a dissipative inequality. Indeed, defining a weighting function $\tilde{g}$ exactly as in [3, p. 500] and the corresponding weighted inner product $\langle , \rangle_{\tilde{g}}$ on Z, one can show without difficulty that (H1) implies the dissipative inequality (see [4, p. 71]) for the nonlinear operator A

(2.4) $\quad \langle Az-Aw, z-w\rangle_{\tilde{g}} \le \omega \langle z-w, z-w\rangle_{\tilde{g}}$

for all $z,w \in D(A)$.

Turning next to the approximation of (2.1) through approximation of (2.3), we let Z^N be the spline subspaces of Z discussed in detail in [3]. For the sake of brevity we here outline the results for the piecewise linear subspaces Z_1^N (see §4 of [3]) given by $Z_1^N = \{(\phi(0),\phi) \mid \phi$ is a continuous first-order spline function with knots at $t_j^N = -jr/N$, $j=0,1,\ldots,N\}$. A careful study of the arguments behind our presentation reveals that the approximation

results given here hold for general spline approximations. For example, if one were to treat cubic spline approximations (Z_3^N of [3]), one would use the appropriate analogues of Theorem 2.5 of [12] and Theorem 21 of [13] (e.g., see Theorem 4.5 of [12]). Hereafter, when we write Z^N, the reader should understand that we mean Z_1^N of [3].

Let $P^N = P_{\tilde{g}}^N$ be the orthogonal projection (in $\langle , \rangle_{\mathcal{g}}$) of Z onto Z^N so that from [3] we have immediately that $P^N z \to z$ for all $z \in Z$. As in [3], we define the approximating operator $A^N = P^N A P^N$ and consider the approximating equations in Z^N given by

$$(2.5) \qquad z^N(t) = P^N\zeta + \int_0^t \{A^N z^N(\sigma) + P^N(g(\sigma),0)\}d\sigma$$

which, because Z^N is finite-dimensional, are equivalent to

$$(2.6) \qquad \dot{z}^N(t) = A^N z^N(t) + P^N(g(t),0)$$

$$z^N(0) = P^N\zeta.$$

From (2.4) and the definition of A^N in terms of the self-adjoint projections P^N, we have at once that under (H1) the sequence $\{A^N\}$ satisfies on Z a uniform dissipative inequality

$$(2.7) \qquad \langle A^N z - A^N w, z-w\rangle_{\mathcal{g}} \le \omega\langle z-w, z-w\rangle_{\mathcal{g}}.$$

Uniqueness of solutions of (2.5) then follows immediately from this inequality. Upon recognition that (2.6) is equivalent to a nonlinear ordinary differential equation in euclidean space with the right-hand side satisfying a global

Lipschitz condition, one can easily argue existence of solutions for (2.6) and hence for (2.5) on any finite interval $[0,T]$. Our main result of this section, which insures that solutions of (2.6) converge to those of (2.1), can now be stated.

Theorem 2.2. Assume (H1), (H2). Let $\zeta = (\phi(0),\phi)$, $\phi \in H^1$ and $g \in H^0(0,T)$ be given, with z^N and x the corresponding solutions on $[0,T]$ of (2.6) and (2.1), respectively. Then $z^N(t) \to y(t) = (x(t;\phi,g), x_t(\phi,g))$, as $N \to \infty$, uniformly in t on $[0,T]$.

Remark 2.2. One can actually obtain slightly stronger results than those given in Theorem 2.2. One can consider solutions of (2.1) and (2.6) corresponding to initial data $(x(0),x_0) = (\eta,\phi) = \zeta$ with $\eta \in R^n$, $\phi \in H^0$ (i.e. $\zeta \in Z$) and argue that the results of Theorem 2.2 hold also in this case.

To indicate briefly our arguments for Theorem 2.2, we consider for given initial data ζ and perturbation g the corresponding solutions z and z^N of (2.3) and (2.5). Defining $\Delta^N(t) \equiv z^N(t)-z(t)$ and $G(t) = (g(t),0)$, we obtain immediately that

$$\Delta^N(t) = (P^N-I)\zeta + \int_0^t \{A^N z^N(\sigma)-Az(\sigma)+(P^N-I)G(\sigma)\}d\sigma. \tag{2.8}$$

We next use a rather standard technique for analysis of differential equations (see [4]), the foundations of which we state as a lemma since we shall refer to it again.

Lemma 2.1. If X is a Hilbert space and $x: [a,b] \to X$ is given by $x(t) = x(a) + \int_a^t y(\sigma)d\sigma$, then $|x(t)|^2 = |x(a)|^2 + 2\int_0^t \langle x(\sigma),y(\sigma)\rangle d\sigma$.

This lemma is essentially a restatement of the well-known result [4, p. 100] that $\frac{d}{dt}\frac{1}{2}|x(t)|^2 = \langle\dot{x}(t),x(t)\rangle$.

Applying Lemma 2.1 to (2.8), we obtain

$$\begin{aligned}
|\Delta^N(t)|^2 &= |(P^N-I)\zeta|^2 + 2\int_0^t \langle A^N z^N(\sigma)-Az(\sigma)+(P^N-I)G(\sigma),\Delta^N(\sigma)\rangle d\sigma \\
&= |(P^N-I)\zeta|^2 + 2\int_0^t \langle A^N z^N(\sigma)-A^N z(\sigma),\Delta^N(\sigma)\rangle d\sigma \\
&\quad + 2\int_0^t \langle (A^N-A)z(\sigma)+(P^N-I)G(\sigma),\Delta^N(\sigma)\rangle d\sigma.
\end{aligned}$$

If we use (2.7) on the first integral term in this last expression, we then have

$$\begin{aligned}
|\Delta^N(t)|^2 &\le |(P^N-I)\zeta|^2 + 2\int_0^t \omega|\Delta^N(\sigma)|^2 d\sigma \\
&\quad + 2\int_0^t \langle (A^N-A)z(\sigma)+(P^N-I)G(\sigma),\Delta^N(\sigma)\rangle d\sigma \\
&\le |(P^N-I)\zeta|^2 + 2\int_0^t \omega|\Delta^N(\sigma)|^2 d\sigma \\
&\quad + 2\int_0^t \{\tfrac{1}{2}|(A^N-A)z(\sigma)|^2 + \tfrac{1}{2}|\Delta^N(\sigma)|^2 + \tfrac{1}{2}|(P^N-I)G(\sigma)|^2 + \tfrac{1}{2}|\Delta^N(\sigma)|^2\}d\sigma \\
&= |(P^N-I)\zeta|^2 + \int_0^t |(A^N-A)z(\sigma)|^2 d\sigma + \int_0^t |(P^N-I)G(\sigma)|^2 d\sigma \\
&\quad + (2\omega+1)\int_0^t |\Delta^N(\sigma)|^2 d\sigma.
\end{aligned}$$

An application of Gronwall's inequality to this then yields the estimate

$$(2.9) \qquad |\Delta^N(t)|^2 \le \{\varepsilon_1(N) + \varepsilon_2(N) + \varepsilon_3(N)\}e^{2(\omega+1)t}$$

where

$$\varepsilon_1(N) = |(P^N - I)\zeta|^2,$$

$$\varepsilon_2(N) = \int_0^T |(A^N - A)z(\sigma)|^2 d\sigma,$$

$$\varepsilon_3(N) = \int_0^T |(P^N - I)G(\sigma)|^2 d\sigma.$$

Since $P^N \to I$ strongly in Z and the convergence $|(P^N - I)G(\sigma)| \to 0$ in ε_3 is dominated, to prove Theorem 2.2 it suffices to argue that $\varepsilon_2(N) \to 0$ as $N \to \infty$. To that end, we state the following sequence of lemmas.

<u>Lemma 2.2</u>. Assume (H1) and let $\mathcal{D} \equiv \{z = (\phi(0), \phi) \,|\, \phi \in H^2\}$. Then $A^N z \to Az$ as $N \to \infty$ for each $z \in \mathcal{D}$.

<u>Lemma 2.3</u>. Let $\mathcal{J} \equiv \{(\zeta, g) \in W \times H^0(0,T) \,|\, \phi \in H^2,\ g \in H^1$, with $\dot{\phi}(0) = F(0,0) + g(0)$ where $\zeta = (\phi(0), \phi)\}$. Assume that (H1), (H2) obtain. Then for $(\zeta, g) \in \mathcal{J}$ the corresponding solution $\sigma \to z(\sigma) = (x(\sigma), x_\sigma)$ of (2.3) (x is the solution of (2.1)) satisfies $z(\sigma) \in \mathcal{D}$ for each $\sigma \in [0,T]$.

<u>Lemma 2.4</u>. Assume (H1), (H2) and let $(\zeta, g) \in \mathcal{J}$ with z^N and z the corresponding solutions of (2.5) and (2.3). Then $z^N(t) \to z(t)$ uniformly in t on $[0,T]$.

Lemma 2.5. Assume (H1). Then the solutions of (2.3) and (2.5) depend continuously (in the $Z \times H^0$ topology) on (ζ, g) in $W \times H^0$, uniformly in t on $[0,T]$.

Lemma 2.6. The set $\mathscr{D}$ defined in Lemma 2.3 is dense in $Z \times H^0$.

We obtain the convergence of Theorem 2.2 by combining Lemmas 2.4, 2.5 and 2.6. The proof of Lemma 2.5 employs Lemma 2.1 along with Gronwall's inequality in much the same way as we did above in deriving (2.9) from (2.8). We note that Lemma 2.3 requires hypothesis (H2) (this is the only place in which it is used) in order to obtain enough smoothness of solutions z of (2.3) so that $z(\sigma) \in \mathscr{D}$ for each σ.

In developing the estimates to establish Lemma 2.4 (which, by our above remarks, requires only that we argue $\varepsilon_2(N) \to 0$), we use heavily the standard spline estimates found in [12] and [13]. Lemmas 2.2 and 2.3 yield that $A^N z(\sigma) \to Az(\sigma)$ for each σ so that to prove Lemma 2.4 one only need show that this convergence is dominated, thereby guaranteeing $\varepsilon_2(N) \to 0$. In making the arguments for Lemma 2.4, one obtains at the same time error estimates on the convergence in Theorem 2.2. For example, one readily finds the following: For $\phi \in H^2$, f satisfying (H1), (H2), $\dot{\phi}(0) = F(0,0)$ and $g \equiv 0$, the convergence $z^N(t) \to z(t)$ is $O(1/N)$. For higher-order splines and higher-order convergence estimates (e.g. cubic splines with convergence $O(1/N^3)$), one of course needs additional smoothness (beyond (H2)) on f.

The convergence given in Theorem 2.2 yields state approximation techniques for nonlinear FDE systems based on the spline methods developed in [3]. These results can be applied to control and identification problems, the latter of which we discuss in the next section.

3. Parameter Identification Problems

Consider now the problem of finding a vector parameter q in some given compact set $Q \subset R^k$ so as to minimize

$$(3.1) \qquad J(q) = \sum_{i=1}^{m} |Cx(t_i;q)-\xi_i|^2$$

subject to

$$(3.2) \qquad \dot{x}(t) = f(q,x(t),x_t,x(t-\tau_1),\ldots,x(t-\tau_\nu)) + g(t)$$

$$x_0 = \phi.$$

Here $\xi_1,\ldots,\xi_m$ are "observations" for $c(t;q) = Cx(t;q)$ at times $t_1,\ldots,t_m$ in $[0,T]$, where C is a given s×n matrix. We assume that $f\colon Q \times Z \times R^{n\nu} \to R^n$ is continuous (in all variables) and satisfies (H1) and (H2) with the same Lipschitz constant K valid in (H1) for all $q \in Q$. For each $q \in Q$, we define, as in §2, a function $F(q)\colon R^n \times C(-r,0;R^n) \to R^n$ by $F(q,\eta,\psi) = f(q,\eta,\psi,\psi(-\tau_1),\ldots,\psi(-\tau_\nu))$ and operators $A(q)(\psi(0),\psi) = (F(q,\psi(0),\psi),D\psi)$ on $D(A(q)) = W$. With the projections defined in §2 we define the sequence of approximating operators $A^N(q)$ by $A^N(q) = P_{\mathcal{G}}^N A(q) P_{\mathcal{G}}^N$. Then, taking $\zeta = (\phi(0),\phi)$, we consider the systems

$$(3.3) \qquad z(t;q) = \zeta + \int_0^t \{A(q)z(\sigma) + (g(\sigma),0)\}d\sigma$$

and

$$(3.4) \qquad z^N(t;q) = P^N\zeta + \int_0^t \{A^N(q)z^N(\sigma) + P^N(g(\sigma),0)\}d\sigma.$$

We may then consider the approximating sequence of identification problems of minimizing over Q the error

$$(3.5) \qquad J^N(q) = \sum_{i=1}^{m} |Cx^N(t_i;q)-\xi_i|^2$$

where $z^N(t;q) = (x^N(t;q), y^N(t;q))$ is the solution of (3.4). Using modifications of the ideas and results of §2, we can then prove the following theorem.

Theorem 3.1. Let q^N be a solution of the problem of minimizing (3.5) over Q. Then $\{q^N\}$ possesses a subsequence $\{q^{N_j}\}$ *which converges to some* $\hat{q}$ in Q which is a solution of the problem of minimizing (3.1) subject to (3.2). If this latter problem has a unique solution $\bar{q}$, then the sequence $\{q^N\}$ itself converges to $\bar{q}$.

Since $A(q)$ and $A^N(q)$ satisfy (2.4) and (2.7) uniformly in $q \in Q$, we have from Theorem 2.2 that $z^N(t;q) \to z(t;q)$ for each $q \in Q$. Then if we choose a subsequence $\{q^{N_j}\}$ of the sequence $\{q^N\}$ of the theorem above with $q^{N_j} \to \hat{q} \in Q$ (Q is compact), we have that for any $q \in Q$

$$(3.6) \qquad J^{N_j}(q^{N_j}) \le J^{N_j}(q).$$

But $J^{N_j}(q) \to J(q)$ and if we establish that $z^{N_j}(t;q^{N_j}) \to z(t;\hat{q})$ whenever $q^{N_j} \to \hat{q}$, we find, upon taking the limit in (3.6), that $J(\hat{q}) \le J(q)$, so that $\hat{q}$ is a minimizer for (3.1), (3.2).

Thus to establish Theorem 3.1, one only need argue that $q^N \to \hat{q}$ implies $z^N(t;q^N) \to z(t;\hat{q})$. This can be done by following almost without change the

sequence of arguments behind Lemmas 2.2 - 2.5 above. Indeed, defining the approximating operators $\mathscr{A}^N = A^N(q^N) = P^N A(q^N)P^N$ and considering the equations

$$(3.7)\qquad z(t;\hat{q}) = \zeta + \int_0^t \{A(\hat{q})z(\sigma)+(g(\sigma),0)\}d\sigma$$

and

$$(3.8)\qquad z^N(t;q^N) = P^N\zeta + \int_0^t \{\mathscr{A}^N z^N(\sigma)+P^N(g(\sigma),0)\}d\sigma,$$

one finds that $\mathscr{A}^N$ satisfies the uniform dissipativeness condition (2.7) and that Lemmas 2.2, 2.3 and 2.4 hold for $\mathscr{A}^N$, $A(\hat{q})$ and (3.7), (3.8) in place of A^N, A and (2.3), (2.5).

We have tested numerically procedures based on the above ideas and present next a sample of our numerical findings.

Example 3.1. Consider the example

$$\dot{x}(t) = a_0x(t) + a_1x(t-1) + \frac{a_2x(t-2)}{1+x(t-2)}$$

$$x_0 \equiv 1,$$

which can be integrated (numerically) using the step method to obtain values $\xi_i = x(t_i)$ to be used as "observations" in (3.1) and (3.5). This was done with the "true" values of $a_0 = 2.0$, $a_1 = 5.0$, $a_2 = 3.0$ and we then used the above methods to identify the parameters a_i (the author wishes to express his appreciation to P. Daniel for her efforts in developing the software packages

used for these and numerous other computations employing spline methods for parameter estimation). At each value of N, a standard least-squares IMSL package based on the Levenberg-Marquardt algorithm was used to find a minimum for (3.5) with $q = (a_1, a_2)$ or $q = (a_0, a_2)$. Our findings are reported in tabular form below. We note that the convergence $q^N \to \hat{q}$ is clearly second-order.

Table 3.1.1: Identify a_1 and a_2 with start-up values $a_{1,0} = 4.0$, $a_{2,0} = 2.0$ for each value of N.

N	a_1^N	a_2^N
2	7.006	2.036
4	5.499	2.796
8	5.126	2.950
16	5.032	2.987
32	5.008	2.996

TRUE VALUES: $a_1 = 5.0$, $a_2 = 3.0$

Table 3.1.2: Identify a_0 and a_2 with start-up values $a_{0,0} = .5$, $a_{2,0} = 4.5$ for each value of N.

N	a_0^N	a_2^N
2	2.158	3.645
4	2.042	3.193
8	2.011	3.050
16	2.003	3.012
32	2.0007	3.0026

TRUE VALUES: $a_0 = 2.0$, $a_2 = 3.0$

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